

**ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ ΚΑΙ ΠΟΛΙΤΙΣΜΟΥ**  
**ΔΙΕΥΘΥΝΣΗ ΑΝΩΤΕΡΗΣ ΚΑΙ ΑΝΩΤΑΤΗΣ ΕΚΠΑΙΔΕΥΣΗΣ**  
**ΥΠΗΡΕΣΙΑ ΕΞΕΤΑΣΕΩΝ**

**ΠΑΓΚΥΠΡΙΕΣ ΕΞΕΤΑΣΕΙΣ**

**2008**

**ΤΕΧΝΟΛΟΓΙΑ Ι ΘΕΩΡΗΤΙΚΗΣ ΚΑΤΕΥΘΥΝΣΗΣ**

**ΜΑΘΗΜΑ : ΜΗΧΑΝΙΚΗ ΚΑΙ ΚΑΤΑΣΚΕΥΕΣ**

**ΗΜΕΡΟΜΗΝΙΑ : ΤΕΤΑΡΤΗ, 4 ΙΟΥΝΙΟΥ 2008**

**ΩΡΑ : 07.30 - 10.00**

**Επιτρεπόμενη διάρκεια γραπτού 2,5 ώρες (150 λεπτά)**

**ΛΥΣΗ ΔΟΚΙΜΙΟΥ**

**ΜΕΡΟΣ Α΄ - Κάθε ορθή απάντηση βαθμολογείται με 4 μονάδες**

1.

$$F_{cr} = \frac{\pi^2 \cdot E \cdot I_{\min}}{\ell^2} = \frac{3,14^2 \cdot 200 \text{ kN} \cdot 20\,000 \text{ mm}^2}{\text{mm}^2 \cdot 16 \cdot 10^6 \text{ mm}^2} = \mathbf{2,46 \text{ kN}}$$

2.  $\Sigma M_A = 0$

$$+20 + 10 \cdot 4 - R_B \cdot 3 = 0$$

$$R_B = \frac{60}{3} = \mathbf{20 \text{ kN}}$$

$\Sigma M_B = 0$

$$+20 + 10 \cdot 1 + R_A \cdot 3 = 0$$

$$+ R_A = \mathbf{-10 \text{ kN}}$$

3.

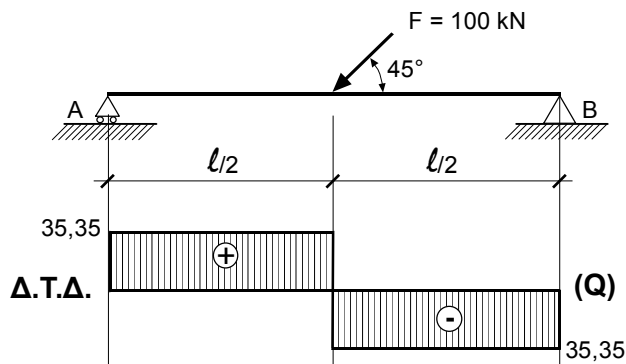
$$I_{\min} = \frac{10^3 \cdot 12}{12} = 1000 \text{ cm}^4$$

$$i_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{1000}{120}} = 2,89 \text{ cm}$$

$$\lambda = \frac{\ell}{i_{\min}} = \frac{0,5 \cdot 300 \text{ cm}}{2,89} = \mathbf{51,9}$$

4.  $F_x = 100 \cdot 0,707 = 70,7 \text{ kN}$

$$R_A = R_B = 35,35 \text{ kN}$$



-3-

$$5. I_{y-y} = \frac{2 \cdot 6^3}{12} + \frac{6 \cdot 1^3}{12} = 36,5 \text{ cm}^4$$

$$6. \Sigma M_A = 0 \quad -30 \cdot 1 + 30 \cdot 4 - R_B \cdot 3 + R \cdot 1,5 = 0 \\ -30 + 120 - 60 \cdot 3 + (q \cdot 3) \cdot 1,5 = 0 \\ q = 20 \text{ kN/m}$$

$$7. \text{Μας δείχνει την αντοχή της δοκού σε κάμψη. } W = \frac{b \cdot h^2}{6} = \frac{20 \cdot 30^2}{6} = 3000 \text{ cm}^3$$

$$8. M_{\max} = \frac{q \cdot \ell^2}{8} \quad 40 = \frac{q \cdot 4^2}{8} \quad q = 20 \text{ kN/m}$$

$$9. \Sigma M_A = 0 \quad -80 \cdot 4 + R_B \cdot 4 = 0 \quad R_B = 80 \text{ kN} \quad R_H = 80 \text{ kN} \\ \Sigma M_B = 0 \quad -80 \cdot 4 - R_A \cdot 4 = 0 \quad R_A = -80 \text{ kN}$$

10. Μεγαλύτερη ροπή αδράνειας  $I_{x-x}$  έχει η διατομή (β) διότι

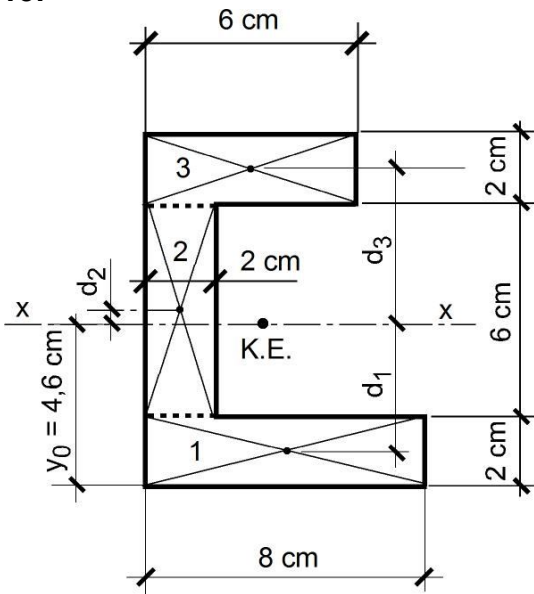
$$\left. \begin{aligned} I_{x(\alpha)} &= \frac{\alpha \cdot \alpha^3}{12} - \frac{\beta \cdot \gamma^3}{12} \\ I_{x(\beta)} &= \frac{\alpha \cdot \alpha^3}{12} - \frac{\gamma \cdot \beta^3}{12} \end{aligned} \right\} \begin{aligned} I_{x(\beta)} &> I_{x(\alpha)} \\ \text{διότι } \frac{\beta \cdot \gamma^3}{12} &> \frac{\gamma \cdot \beta^3}{12} \end{aligned}$$

$$11. M_A = -10 \cdot 2 - 5 \cdot 1 = -25 \text{ kN} \cdot \text{m} \\ M_B = -10 \cdot 1 = -10 \text{ kN} \cdot \text{m}$$

12. Δ.Τ.Δ. → (β)  
Δ.Ρ.Κ. → (στ)

ΜΕΡΟΣ Β'

13.



$$I_{x-x} = \left( \frac{8 \cdot 2^3}{12} + 2 \cdot 8 \cdot 3,6^2 \right) + \left( \frac{2 \cdot 6^3}{12} + 2 \cdot 6 \cdot 0,4^2 \right) + \left( \frac{6 \cdot 2^3}{12} + 2 \cdot 6 \cdot 4,4^2 \right) =$$

$$= (5,33 + 207,36) + (36 + 1,92) + (4 + 232,32) = 486,93 \text{ cm}^4$$

$$d_1 = 4,6 - 1 = 3,6 \text{ cm}$$

$$d_2 = 5 - 4,6 = 0,4 \text{ cm}$$

$$d_3 = 9 - 4,6 = 4,4 \text{ cm}$$

14.  $\frac{M}{I} = \frac{\sigma}{y}$       $I_{x-x} = \frac{4 \cdot 6^3}{12} = 72 \text{ cm}^4$       $\frac{M}{72 \text{ cm}^4} = \frac{150 \text{ N}}{\text{mm}^2 \cdot 30 \text{ mm}}$

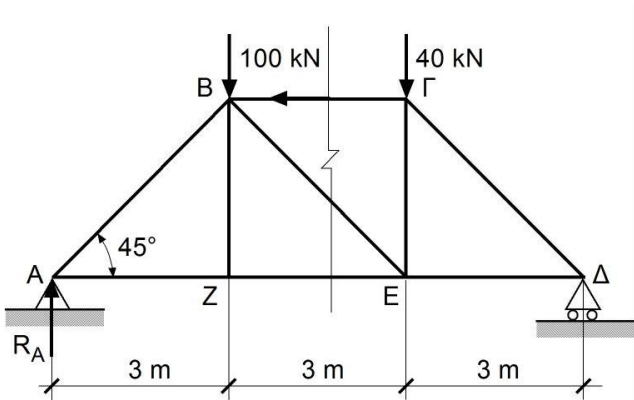
$$\frac{M}{72 \cdot 10^4 \text{ mm}^4} = \frac{150 \text{ N}}{30 \text{ mm}^3} \quad M = 360 \cdot 10^4 \text{ Nmm}$$

$$M = 3600 \text{ kN} \cdot \text{mm} = M_{\Gamma} = 3,6 \text{ kN} \cdot \text{m}$$

$$M_{\Gamma} = R_A \cdot 2 \quad 3,6 \text{ kN} \cdot \text{m} = R_A = 1,8 \text{ kN}$$

$$\Sigma M_B = 0 \quad R_A \cdot 5 - F \cdot 3 = 0 \quad 1,8 \cdot 5 - F \cdot 3 = 0 \quad F = 3 \text{ kN}$$

15.



$$\Sigma M_A = 0 \quad 100 \cdot 3 + 40 \cdot 6 - R_{\Delta} \cdot 9 = 0$$

$$540 = R_{\Delta} \cdot 9 \quad R_{\Delta} = 60 \text{ kN}$$

$$\Sigma M_B = 0 \quad R_A \cdot 9 - 100 \cdot 6 - 40 \cdot 3 = 0$$

$$R_A \cdot 9 = 720 \quad R_A = 80 \text{ kN}$$

$$\Sigma M_E = 0 \quad R_A \cdot 6 - 100 \cdot 3 - F_{(B\Gamma)} \cdot 3 = 0$$

$$80 \cdot 6 - 300 = F_{(B\Gamma)} \cdot 3 \quad F_{(B\Gamma)} = 60 \text{ kN}$$

16.  $I_{\text{MIN}} = \frac{40 \cdot 30^3}{12} = 90000 \text{ cm}^4 \quad \ell = 0,7L$

$$F_{\text{cr}} = \frac{\pi^2 E \cdot I_{\text{min}}}{\ell^2} \quad \ell^2 = \frac{\pi^2 \cdot E \cdot I_{\text{min}}}{F_{\text{cr}}}$$

$$\ell = \pi \sqrt{\frac{E \cdot I_{\text{min}}}{F_{\text{cr}}}} = 3,14 \sqrt{\frac{200 \text{ kN} \cdot 90000 \text{ mm}^4}{\text{mm}^2 \cdot 450 \text{ kN}}} = 3,14 \sqrt{40000 \text{ mm}^2}$$

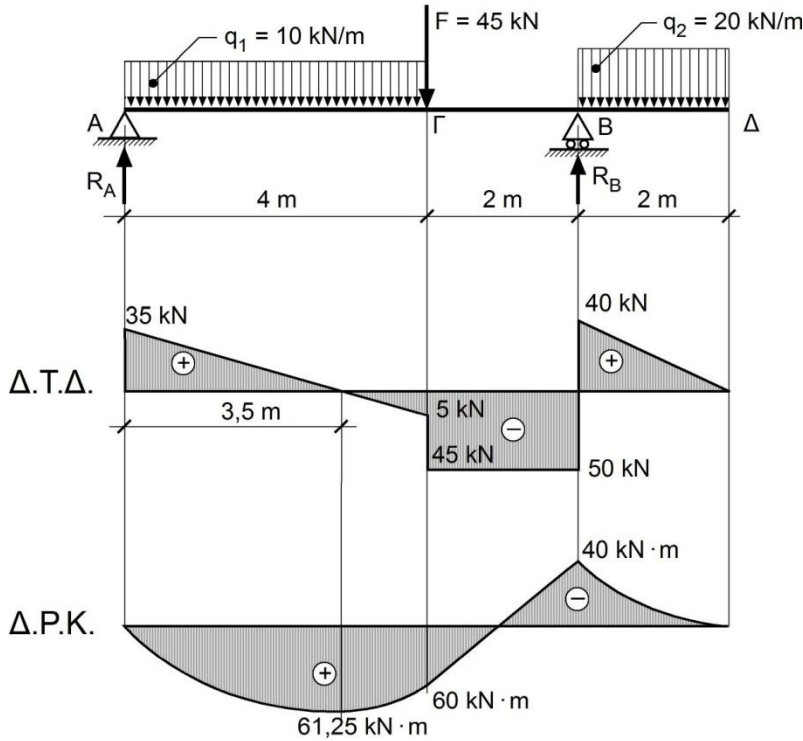
$$\ell = 3,14 \sqrt{\frac{200 \text{ kN} \cdot 90000 \cdot 10^4 \text{ mm}^4}{\text{mm}^2 \cdot 450 \text{ kN}}}$$

$$\ell = 3,14 \cdot 20000 = 62800 \text{ mm} = 62,8 \text{ m}$$

$$\ell = 0,7L \quad L = \frac{\ell}{0,7} = \frac{62,8}{0,7} = 89,71 \text{ m}$$

ΜΕΡΟΣ Γ΄

17.



$$\begin{aligned} \Sigma M_A = 0 : R_1 \cdot 2 + F \cdot 4 + R_2 \cdot 7 - R_B \cdot 6 &= 0 \\ 40 \cdot 2 + 45 \cdot 4 + 40 \cdot 7 &= R_B \cdot 6 \\ 80 + 180 + 280 &= R_B \cdot 6 \\ R_B &= 90 \text{ kN} \end{aligned}$$

$$\begin{aligned} \Sigma M_B = 0 : R_A \cdot 6 + R_2 \cdot 1 - R_1 \cdot 4 - F \cdot 2 &= 0 \\ R_A \cdot 6 &= 40 \cdot 4 + 45 \cdot 2 - 40 \cdot 1 \\ R_A \cdot 6 &= 160 + 90 - 40 \\ R_A &= 35 \text{ kN} \end{aligned}$$

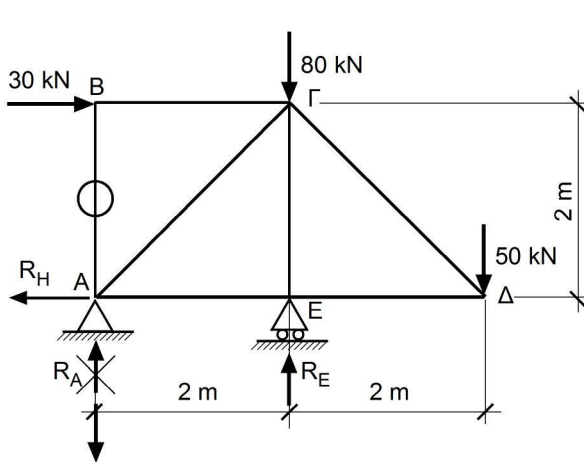
$$\frac{35}{5} = \frac{x}{4-x}$$

$$\begin{aligned} 35(4-x) &= 5x \\ 140 - 35x &= 5x \\ 140 &= 40x \\ x &= 3,5\text{m} \end{aligned}$$

$$\begin{aligned} \text{Τομή στο σημείο όπου } Q &= 0 \\ M_{\max} &= 35 \times 3,5 - R_x \cdot 1,75 \\ 122,5 - 61,25 &= 61,25 \text{ kN} \cdot \text{m} \end{aligned}$$

$$\begin{aligned} \text{Τομή στο σημείο } \Gamma \\ M_{\Gamma} &= 35 \cdot 4 - 40 \cdot 2 \\ 140 - 80 &= 60 \text{ kN} \cdot \text{m} \end{aligned}$$

18.



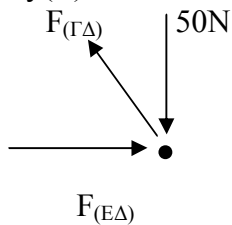
$$\begin{aligned} \Sigma M_A = 0 : 30 \cdot 2 + 80 \cdot 2 + 50 \cdot 4 - R_E \cdot 2 &= 0 \\ 60 + 160 + 200 &= R_E \cdot 2 \\ 420 &= R_E \cdot 2 \\ R_E &= 210 \text{ N} \end{aligned}$$

$$\begin{aligned} \Sigma M_E = 0 : R_A \cdot 2 + 30 \cdot 2 + 50 \cdot 2 &= 0 \\ R_A \cdot 2 &= -60 - 100 \\ R_A &= -80 \text{ N} \end{aligned}$$

| Ράβδος     | Θλίψη | Εφελκ. |
|------------|-------|--------|
| $F_{(AB)}$ | -     | -      |
| $F_{(BΓ)}$ | 30    | -      |
| $F_{(ΓΔ)}$ | -     | 70,72  |
| $F_{(EΔ)}$ | 50    | -      |
| $F_{(AE)}$ | 50    | -      |
| $F_{(EΓ)}$ | 210   | -      |
| $F_{(AΓ)}$ | -     | 113,15 |

$F_{(AB)}=0$

Κόμβος (Δ)

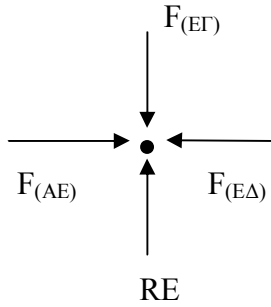


$$\begin{aligned} \Sigma f_y = 0 \\ -50 + F_{(ΓΔ)} \cdot 0.707 &= 0 \\ F_{(ΓΔ)} &= \frac{50}{0.707} = 70.72 \text{ N} \end{aligned}$$

$$\Sigma F_x = 0 \quad F_{(EΔ)} - F_{(ΓΔ)} \cdot 0,707 = 0$$

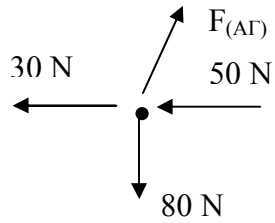
$$F_{(EΔ)} = 50 \text{ N}$$

Κόμβος (E)



$$\begin{aligned} F_{(AE)} &= F_{(E\Delta)} \\ F_{(AE)} &= 50 \text{ N} \\ F_{(E\Gamma)} &= RE \\ F_{(E\Gamma)} &= 210 \text{ N} \end{aligned}$$

Κόμβος (A)



$$\Sigma F_y = 0$$

$$F_{(A\Gamma)} \cdot 0,707 - 80 = 0$$

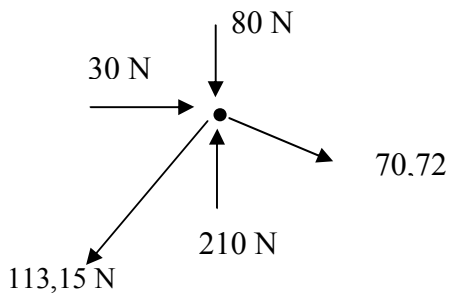
$$F_{(A\Gamma)} = \frac{80}{0,707} = 113,15 \text{ N}$$

Κόμβος (B)

$$F_{(B\Gamma)} = 30 \text{ N} = F_1$$

Επαλήθευση

Κόμβος (Γ)





$$\Sigma F_x = 0$$

$$30 + 70,72 \cdot 0,707 - 113,15 \cdot 0,707 = 0$$

$$30 + 50 - 80 = 0$$

$$\Sigma F_y = 0$$

$$+ 210 - 80 - 70,72 \cdot 0,707 - 113,15 \cdot 0,707 = 0$$

$$+ 210 - 80 - 50 - 80 = 0$$

$$210 - 210 = 0$$